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ABSTRACT

In Chemical Graph Theory, several degree based topological indices were introduced and studied since 1972. In this paper, we compute the first, second, third, fourth and fifth multiplicative arithmetic-geometric indices of some important chemical drugs which appeared in medical science.

KEYWORDS: multiplicative arithmetic-geometric index, nanostructure.

Mathematics Subject Classification: 05C05, 05C07, 05C90.

1. INTRODUCTION

Let G be a finite, simple connected graph with a vertex set $V(G)$ and an edge set $E(G)$. The degree $d(v)$ of a vertex v is the number of vertices adjacent to v . We refer to [1] for undefined term and notation.

A molecular graph is a simple graph related to the structure of a chemical compound. Each vertex of this graph represents an atom of the molecule and its edges to the bonds between atoms. A topological index is a numeric quantity from structural graph of a molecule. These indices are useful for establishing correlation between the structures of a molecular compound and its physicochemical properties.

Very recently Kulli [2] introduced the first multiplicative arithmetic-geometric index of a graph G and it is defined as

$$AG_1II(G) = \prod_{uv \in E(G)} \frac{d(u) + d(v)}{2\sqrt{d(u)d(v)}}$$

Many other multiplicative indices were studied, for example, in [3, 4, 5, 6, 7, 8, 9, 10].

Motivated by the definition of the first multiplicative arithmetic-geometric index and by previous research on topological indices, Kulli proposed the second, third, fourth and fifth multiplicative arithmetic-geometric indices [11] of a graph as follows:

The second multiplicative arithmetic-geometric index of a graph G is defined as

$$AG_2II(G) = \prod_{uv \in E(G)} \frac{n(u) + n(v)}{2\sqrt{n(u)n(v)}}$$

where the number $n(u)$ of vertices of G lying closer to the vertex u than to the vertex v for the edge uv of a graph G .

The third multiplicative arithmetic-geometric index of a graph G is defined as

$$AG_3II(G) = \prod_{uv \in E(G)} \frac{m(u) + m(v)}{2\sqrt{m(u)m(v)}}$$

Where the number $m(u)$ of edges of G lying closer to the vertex u than to the vertex v for the edge uv of a graph G .

The fourth multiplicative arithmetic-geometric index of a graph G is defined as

$$AG_4H(G) = \prod_{uv \in E(G)} \frac{\varepsilon(u) + \varepsilon(v)}{2\sqrt{\varepsilon(u)\varepsilon(v)}}$$

where the number $\varepsilon(u)$ is the eccentricity of vertex u .

The fifth multiplicative arithmetic-geometric index of a graph G is defined as

$$AG_5H(G) = \prod_{uv \in E(G)} \frac{s(u) + s(v)}{2\sqrt{s(u)s(v)}}$$

where $s(u)$ denote the sum of the degrees of all vertices adjacent to a vertex u .

Recently, some new versions of topological indices were studied [12, 13, 14, 15, 16].

In this paper, the first, second, third, fourth and fifth multiplicative arithmetic-geometric Zagreb indices of some important molecular structures such as chloroquine, hydrochloroquine, remdesivir are computed. For chemical drugs, see [17, 18].

2. RESULTS AND DISCUSSION: CHLOROQUINE

Chloroquine is an antiviral compound (drug) which was discovered in 1934 by H.Andersag. This drug is medication primarily used to prevent and treat malaria.

Let G_1 be the chemical structure of chloroquine. This structure has 21 atoms and 23 bonds, see Figure 1.

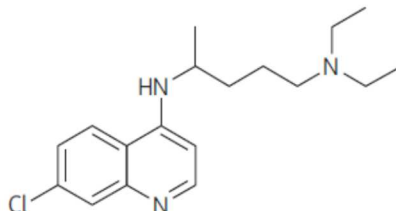


Figure 1. Chemical structure of chloroquine

From Figure 1, we obtain that

- (i) $\{(d(u), d(v)) \setminus uv \in E(G_1)\}$ has 5 bond set partitions,
- (ii) $\{(n(u), n(v)) \setminus uv \in E(G_1)\}$ has 10 bond set partitions,
- (iii) $\{(m(u), m(v)) \setminus uv \in E(G_1)\}$ has 12 bond set partitions,
- (iv) $\{(\varepsilon(u), \varepsilon(v)) \setminus uv \in E(G_1)\}$ has 7 bond set partitions,
- (iv) $\{(s(u), s(v)) \setminus uv \in E(G_1)\}$ has 10 bond set partitions.

Figure 1. Chemical structure of chloroquine

| | | | | | | |
|--------------------------------------|--------|--------|--------|--------|---------|---------|
| $d(u), d(v) \setminus uv \in E(G_1)$ | (1, 2) | (1,3) | (2, 2) | (2, 3) | (3, 3) | |
| Number of bonds | 2 | 2 | 5 | 12 | 2 | |
| $n(u), n(v) \setminus uv \in E(G_1)$ | (1,19) | (1,20) | (2,18) | (3,17) | (4,16) | |
| Number of bonds | 2 | 4 | 2 | 4 | 1 | |
| | (5,15) | (6,14) | (7,13) | (9,11) | (10,10) | |
| | 4 | 1 | 3 | 1 | 1 | |
| $m(u), m(v) \setminus uv \in E(G_1)$ | (1,21) | (1,22) | (2,19) | (3,18) | (4,17) | (5,15) |
| Number of bonds | 2 | 4 | 2 | 4 | 1 | 3 |
| | (5,16) | (6,15) | (7,14) | (8,13) | (9,13) | (10,12) |

| | | | | | | |
|--|---------|---------|-------|--------|---------|---|
| | 1 | 1 | 2 | 1 | 1 | 1 |
| $\varepsilon(u), \varepsilon(v) \setminus uv \in E(G_1)$ | (7,7) | (8,7) | (8,9) | (9,10) | (10,11) | |
| Number of bonds | 1 | 3 | 3 | 4 | 5 | |
| | (11,12) | (12,13) | | | | |
| | 4 | 3 | | | | |
| $s(u), s(v) \setminus uv \in E(G_1)$ | (2,4) | (3,5) | (4,5) | (4,6) | (5,5) | |
| Number of bonds | 2 | 2 | 4 | 2 | 3 | |
| | (5,6) | (5,7) | (5,8) | (6,7) | (7,8) | |
| | 3 | 2 | 1 | 2 | 2 | |

In the following theorem, we compute the different versions of multiplicative arithmetic-geometric indices of chloroquine.

Theorem 1. Let G_1 be the chemical structure of chloroquine. Then

$$(i) \quad AG_1II(G_1) = \left(\frac{3}{2\sqrt{2}}\right)^2 \times \left(\frac{2}{\sqrt{3}}\right)^2 \times \left(\frac{5}{2\sqrt{6}}\right)^{12}.$$

$$(ii) \quad AG_2II(G_1) = \left(\frac{10}{\sqrt{19}}\right)^2 \times \left(\frac{21}{4\sqrt{5}}\right)^4 \times \left(\frac{10}{\sqrt{51}}\right)^4 \times \left(\frac{2}{\sqrt{3}}\right)^4 \times \left(\frac{5}{\sqrt{21}}\right)^1 \\ \times \left(\frac{10}{\sqrt{91}}\right)^3 \times \left(\frac{10}{3\sqrt{11}}\right)^1 \times \left(\frac{125}{36}\right)^1$$

$$(iii) \quad AG_3II(G_1) = \left(\frac{11}{\sqrt{21}}\right)^2 \times \left(\frac{23}{2\sqrt{22}}\right)^4 \times \left(\frac{21}{2\sqrt{38}}\right)^2 \times \left(\frac{7}{2\sqrt{6}}\right)^4 \times \left(\frac{21}{4\sqrt{17}}\right)^1 \times \left(\frac{2}{\sqrt{3}}\right)^3 \\ \times \left(\frac{21}{8\sqrt{5}}\right)^1 \times \left(\frac{7}{2\sqrt{10}}\right)^1 \times \left(\frac{3}{2\sqrt{2}}\right)^2 \times \left(\frac{21}{4\sqrt{26}}\right)^1 \times \left(\frac{11}{3\sqrt{13}}\right)^1 \times \left(\frac{11}{2\sqrt{30}}\right)^1$$

$$(iv) \quad AG_4II(G_1) = \left(\frac{15}{4\sqrt{14}}\right)^3 \times \left(\frac{17}{12\sqrt{2}}\right)^3 \times \left(\frac{19}{6\sqrt{10}}\right)^4 \times \left(\frac{21}{2\sqrt{110}}\right)^5 \times \left(\frac{23}{4\sqrt{33}}\right)^4 \times \left(\frac{25}{4\sqrt{39}}\right)^3.$$

$$(v) \quad AG_5II(G_1) = \left(\frac{3}{2\sqrt{2}}\right)^2 \times \left(\frac{4}{\sqrt{15}}\right)^2 \times \left(\frac{9}{4\sqrt{5}}\right)^4 \times \left(\frac{5}{2\sqrt{6}}\right)^2 \times \left(\frac{11}{2\sqrt{30}}\right)^3 \\ \times \left(\frac{6}{\sqrt{35}}\right)^2 \times \left(\frac{13}{4\sqrt{10}}\right) \times \left(\frac{13}{2\sqrt{42}}\right)^2 \times \left(\frac{15}{4\sqrt{14}}\right)^2.$$

Proof: By using the definitions and cardinalities of the bond partitions of G_1 , we deduce

$$(i) \quad AG_1II(G_1) = \prod_{uv \in E(G_1)} \frac{d(u) + d(v)}{2\sqrt{d(u)d(v)}} \\ = \left(\frac{1+2}{2\sqrt{1 \times 2}}\right)^2 \times \left(\frac{1+3}{2\sqrt{1 \times 3}}\right)^2 \times \left(\frac{2+2}{2\sqrt{2 \times 2}}\right)^5 \times \left(\frac{2+3}{2\sqrt{2 \times 3}}\right)^{12} \times \left(\frac{3+3}{2\sqrt{3 \times 3}}\right)^2$$

After simplification, we obtain the desired result.

$$(ii) \quad AG_2II(G_1) = \prod_{uv \in E(G_1)} \frac{n(u) + n(v)}{2\sqrt{n(u)n(v)}}$$

$$\begin{aligned}
 &= \left(\frac{1+19}{2\sqrt{1 \times 19}}\right)^2 \times \left(\frac{1+20}{2\sqrt{1 \times 20}}\right)^4 \times \left(\frac{2+18}{2\sqrt{2 \times 18}}\right)^2 \times \left(\frac{3+17}{2\sqrt{3 \times 17}}\right)^4 \times \left(\frac{4+16}{2\sqrt{4 \times 16}}\right)^1 \\
 &\times 4 \left(\frac{5+15}{2\sqrt{5 \times 15}}\right)^4 \times \left(\frac{6+14}{2\sqrt{6 \times 14}}\right)^1 \times 3 \left(\frac{7+13}{2\sqrt{7 \times 13}}\right)^3 \times \left(\frac{9+11}{2\sqrt{9 \times 11}}\right)^1 \times \left(\frac{10+10}{2\sqrt{10 \times 10}}\right)^1.
 \end{aligned}$$

After simplification, we obtain the desired result.

$$\begin{aligned}
 \text{(iii)} \quad AG_3H(G_1) &= \prod_{uv \in E(G_1)} \frac{m(u) + m(v)}{2\sqrt{m(u)m(v)}} \\
 &= \left(\frac{1+21}{2\sqrt{1 \times 21}}\right)^2 \times \left(\frac{1+22}{2\sqrt{1 \times 22}}\right)^4 \times \left(\frac{2+19}{2\sqrt{2 \times 19}}\right)^2 \times \left(\frac{3+18}{2\sqrt{3 \times 18}}\right)^4 \times \left(\frac{4+17}{2\sqrt{4 \times 17}}\right)^1 \times \left(\frac{5+15}{2\sqrt{5 \times 15}}\right)^3 \\
 &\times \left(\frac{5+16}{2\sqrt{5 \times 16}}\right)^1 \times \left(\frac{6+15}{2\sqrt{6 \times 15}}\right)^1 \times \left(\frac{7+14}{2\sqrt{7 \times 14}}\right)^2 \times \left(\frac{8+13}{2\sqrt{8 \times 13}}\right)^1 \times \left(\frac{9+13}{2\sqrt{9 \times 13}}\right)^1 \times \left(\frac{10+12}{2\sqrt{10 \times 12}}\right)^1.
 \end{aligned}$$

After simplification, we obtain the desired result.

$$\begin{aligned}
 \text{(iv)} \quad AG_4H(G_1) &= \prod_{uv \in E(G_1)} \frac{\varepsilon(u) + \varepsilon(v)}{2\sqrt{\varepsilon(u)\varepsilon(v)}} \\
 &= \left(\frac{7+7}{2\sqrt{7 \times 7}}\right) \times \left(\frac{8+7}{2\sqrt{8 \times 7}}\right)^3 \times \left(\frac{8+9}{2\sqrt{8 \times 9}}\right)^3 \times \left(\frac{9+10}{2\sqrt{9 \times 10}}\right)^4 \times \left(\frac{10+11}{2\sqrt{10 \times 11}}\right)^5 \\
 &\times \left(\frac{11+12}{2\sqrt{11 \times 12}}\right)^4 \times \left(\frac{12+13}{2\sqrt{12 \times 13}}\right)^3.
 \end{aligned}$$

After simplification, we obtain the desired result.

$$\begin{aligned}
 \text{(v)} \quad AG_5H(G_1) &= \prod_{uv \in E(G_1)} \frac{s(u) + s(v)}{2\sqrt{s(u)s(v)}} \\
 &= \left(\frac{2+4}{2\sqrt{2 \times 4}}\right)^2 \times \left(\frac{3+5}{2\sqrt{3 \times 5}}\right)^2 \times \left(\frac{4+5}{2\sqrt{4 \times 5}}\right)^4 \times \left(\frac{4+6}{2\sqrt{4 \times 6}}\right)^2 \times \left(\frac{5+5}{2\sqrt{5 \times 5}}\right)^3 \\
 &\times \left(\frac{5+6}{2\sqrt{5 \times 6}}\right)^3 \times \left(\frac{5+7}{2\sqrt{5 \times 7}}\right)^2 \times \left(\frac{5+8}{2\sqrt{5 \times 8}}\right)^1 \times \left(\frac{6+7}{2\sqrt{6 \times 7}}\right)^2 \times \left(\frac{7+8}{2\sqrt{7 \times 8}}\right)^2
 \end{aligned}$$

After simplification, we obtain the desired result.

3. RESULTS AND DISCUSSION: HYDROXYCHLOROQUINE

Hydroxychloroquine is another antiviral compound (drug) which has antiviral activity very similar to that of chloroquine. These compounds have been repurposed for the treatment of a number of other conditions including HIV, systemic lupus erythmatosus and rheumatoid arthritis .

Let G_2 be the chemical structure of hydroxychloroquine. This structure has 22 vertices and 24 edges, see Figure 2.

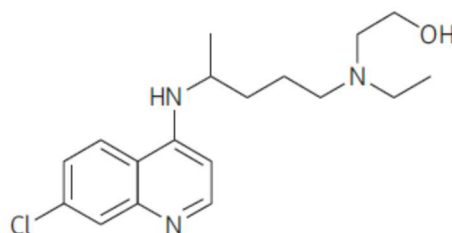


Figure 2. Chemical structure of hydroxychloroquine

From Figure 2, we obtain that

- (i) $\{(d(u), d(v)) \setminus uv \in E(G_2)\}$ has 5 bond set partitions,
- (ii) $\{(n(u), n(v)) \setminus uv \in E(G_2)\}$ has 9 bond set partitions,
- (iii) $\{(m(u), m(v)) \setminus uv \in E(G_2)\}$ has 12 bond set partitions,
- (iv) $\{(\varepsilon(u), \varepsilon(v)) \setminus uv \in E(G_2)\}$ has 7 bond set partitions,
- (iv) $\{(s(u), s(v)) \setminus uv \in E(G_2)\}$ has 11 bond set partition

Table 2. Bond set partitions of hydroxychloroquine

| | | | | | | |
|--|---------|---------|---------|---------|---------|---------|
| $d(u), d(v) \setminus uv \in E(G_2)$ | (1, 2) | (1,3) | (2, 2) | (2, 3) | (3, 3) | |
| Number of bonds | 2 | 2 | 6 | 12 | 2 | |
| $n(u), n(v) \setminus uv \in E(G_2)$ | (1,20) | (1,21) | (2,19) | (3,18) | (5,16) | |
| Number of bonds | 2 | 4 | 3 | 4 | 4 | |
| | (6,15) | (7,14) | (10,11) | (8,13) | | |
| | 3 | 2 | 1 | 1 | | |
| $m(u), m(v) \setminus uv \in E(G_2)$ | (1,22) | (1,23) | (2,20) | (2,21) | (3,19) | (5,16) |
| Number of bonds | 2 | 4 | 2 | 1 | 4 | 3 |
| | (5,17) | (6,16) | (7,15) | (8,14) | (10,13) | (11,12) |
| | 1 | 1 | 1 | 3 | 1 | 1 |
| $\varepsilon(u), \varepsilon(v) \setminus uv \in E(G_2)$ | (7,8) | (8,9) | (9,10) | (10,11) | (11,12) | |
| Number of bonds | 3 | 2 | 3 | 4 | 6 | |
| | (12,13) | (13,14) | | | | |
| | 4 | 2 | | | | |
| $s(u), s(v) \setminus uv \in E(G_2)$ | (2,3) | (2,4) | (3,5) | (4,5) | (4,6) | (5,5) |
| Number of bonds | 1 | 1 | 3 | 4 | 1 | 3 |
| | (5,6) | (5,7) | (5,8) | (6,7) | (7,8) | |
| | 4 | 2 | 1 | 2 | 2 | |

In the following theorem, we compute the different versions of multiplicative arithmetic-geometric indices of hydroxychloroquine.

Theorem 2. Let G_2 be the chemical structure of hydroxychloroquine. Then

- (i) $AG_1II(G_2) = \left(\frac{3}{2\sqrt{2}}\right)^2 \times \left(\frac{2}{\sqrt{3}}\right)^2 \times \left(\frac{5}{2\sqrt{6}}\right)^{12}$.
- (ii) $AG_2II(G_2) = \left(\frac{21}{2\sqrt{20}}\right)^2 \times \left(\frac{11}{\sqrt{21}}\right)^4 \times \left(\frac{21}{2\sqrt{38}}\right)^4 \times \left(\frac{21}{2\sqrt{54}}\right)^4 \times \left(\frac{21}{8\sqrt{5}}\right)^1$
 $\times \left(\frac{21}{6\sqrt{10}}\right)^3 \times \left(\frac{3}{2\sqrt{2}}\right)^2 \times \left(\frac{21}{2\sqrt{110}}\right)^1 \times \left(\frac{21}{2\sqrt{104}}\right)^1$.

$$(iii) AG_3II(G_2) = \left(\frac{23}{2\sqrt{22}}\right)^2 \times \left(\frac{12}{\sqrt{23}}\right)^4 \times \left(\frac{11}{2\sqrt{10}}\right)^4 \times \left(\frac{23}{2\sqrt{42}}\right)^4 \times \left(\frac{11}{\sqrt{57}}\right)^1 \times \left(\frac{21}{8\sqrt{5}}\right)^3 \\ + \left(\frac{11}{\sqrt{85}}\right)^1 \times \left(\frac{11}{4\sqrt{6}}\right)^1 \times \left(\frac{11}{2\sqrt{105}}\right)^1 \times \left(\frac{11}{4\sqrt{7}}\right)^3 \times \left(\frac{23}{2\sqrt{130}}\right)^1 \times \left(\frac{23}{2\sqrt{132}}\right)^1.$$

$$(iv) AG_4II(G_2) = \left(\frac{15}{4\sqrt{14}}\right)^3 \times \left(\frac{17}{12\sqrt{2}}\right)^2 \times \left(\frac{19}{6\sqrt{10}}\right)^3 \times \left(\frac{21}{2\sqrt{110}}\right)^4 \times \left(\frac{23}{4\sqrt{33}}\right)^6 \\ \times \left(\frac{25}{4\sqrt{39}}\right)^4 \times \left(\frac{27}{4\sqrt{182}}\right)^2.$$

$$(v) AG_5II(G_2) = \left(\frac{5}{2\sqrt{6}}\right)^1 \times \left(\frac{3}{2\sqrt{2}}\right)^1 \times \left(\frac{4}{\sqrt{15}}\right)^3 \times \left(\frac{9}{4\sqrt{5}}\right)^4 \times \left(\frac{5}{2\sqrt{6}}\right)^1 \times \left(\frac{11}{2\sqrt{30}}\right)^4 \\ \times \left(\frac{6}{\sqrt{35}}\right)^2 \times \left(\frac{13}{4\sqrt{10}}\right)^1 \times \left(\frac{13}{2\sqrt{42}}\right)^2 \times \left(\frac{15}{4\sqrt{14}}\right)^2.$$

Proof: By using the definitions and cardinalities of the bond partitions of G_2 , we deduce

$$(i) AG_1II(G_2) = \prod_{uv \in E(G_2)} \frac{d(u) + d(v)}{2\sqrt{d(u)d(v)}} \\ = \left(\frac{1+2}{2\sqrt{1 \times 2}}\right)^2 \times \left(\frac{1+3}{2\sqrt{1 \times 3}}\right)^2 \times \left(\frac{2+2}{2\sqrt{2 \times 2}}\right)^6 \times \left(\frac{2+3}{2\sqrt{2 \times 3}}\right)^{12} \times \left(\frac{3+3}{2\sqrt{3 \times 3}}\right)^2.$$

After simplification, we obtain the desired result.

$$(ii) AG_2II(G_2) = \prod_{uv \in E(G_2)} \frac{n(u) + n(v)}{2\sqrt{n(u)n(v)}} \\ = 2 \left(\frac{1+20}{2\sqrt{1 \times 20}}\right)^2 \times 4 \left(\frac{1+21}{2\sqrt{1 \times 21}}\right)^4 \times 3 \left(\frac{2+19}{2\sqrt{2 \times 19}}\right)^2 \times 4 \left(\frac{3+18}{2\sqrt{3 \times 18}}\right)^4 \times 4 \left(\frac{5+16}{2\sqrt{5 \times 16}}\right)^4 \\ \times 3 \left(\frac{6+15}{2\sqrt{6 \times 15}}\right)^4 \times 2 \left(\frac{7+14}{2\sqrt{7 \times 14}}\right)^1 \times \left(\frac{10+11}{2\sqrt{10 \times 11}}\right)^3 \times \left(\frac{8+13}{2\sqrt{8 \times 13}}\right)^1.$$

After simplification, we obtain the desired result.

$$(iii) AG_3II(G_2) = \prod_{uv \in E(G_2)} \frac{m(u) + m(v)}{2\sqrt{m(u)m(v)}} \\ = 2 \left(\frac{1+22}{2\sqrt{1 \times 22}}\right)^2 \times \left(\frac{1+23}{2\sqrt{1 \times 23}}\right)^4 \times \left(\frac{2+20}{2\sqrt{2 \times 20}}\right)^2 \times \left(\frac{2+21}{2\sqrt{2 \times 21}}\right)^1 \times \left(\frac{3+19}{2\sqrt{3 \times 19}}\right)^4 \\ \times \left(\frac{5+17}{2\sqrt{5 \times 17}}\right)^1 \times \left(\frac{6+16}{2\sqrt{6 \times 16}}\right)^1 \times \left(\frac{7+15}{2\sqrt{7 \times 15}}\right)^3 \times \left(\frac{8+14}{2\sqrt{8 \times 14}}\right)^3 \times \left(\frac{10+13}{2\sqrt{10 \times 13}}\right)^1 \times \left(\frac{11+12}{2\sqrt{11 \times 12}}\right)^1.$$

After simplification, we obtain the desired result.

$$(iv) AG_4II(G_2) = \prod_{uv \in E(G_2)} \frac{\varepsilon(u) + \varepsilon(v)}{2\sqrt{\varepsilon(u)\varepsilon(v)}}$$

$$= \left(\frac{7+8}{2\sqrt{7 \times 8}}\right)^3 \times \left(\frac{8+9}{2\sqrt{8 \times 9}}\right)^2 \times \left(\frac{9+10}{2\sqrt{9 \times 10}}\right)^3 \times \left(\frac{10+11}{2\sqrt{10 \times 11}}\right)^4 \times \left(\frac{11+12}{2\sqrt{11 \times 12}}\right)^6$$

$$\times \left(\frac{12+13}{2\sqrt{12 \times 13}}\right)^4 \times \left(\frac{13+14}{2\sqrt{13 \times 14}}\right)^2.$$

After simplification, we obtain the desired result.

$$(v) \quad AG_5H(G_2) = \prod_{uv \in E(G_2)} \frac{s(u) + s(v)}{2\sqrt{s(u)s(v)}}$$

$$= \left(\frac{2+3}{2\sqrt{2 \times 3}}\right)^1 \times \left(\frac{2+4}{2\sqrt{2 \times 4}}\right)^1 \times \left(\frac{3+5}{2\sqrt{3 \times 5}}\right)^3 \times \left(\frac{4+5}{2\sqrt{4 \times 5}}\right)^4 \times \left(\frac{4+6}{2\sqrt{4 \times 6}}\right)^1 \times \left(\frac{5+5}{2\sqrt{5 \times 5}}\right)^3$$

$$\times \left(\frac{5+6}{2\sqrt{5 \times 6}}\right)^4 \times \left(\frac{5+7}{2\sqrt{5 \times 7}}\right)^2 \times \left(\frac{5+8}{2\sqrt{5 \times 8}}\right)^1 \times \left(\frac{6+7}{2\sqrt{6 \times 7}}\right)^2 \times \left(\frac{7+8}{2\sqrt{7 \times 8}}\right)^2.$$

After simplification, we get the desired result.

4. RESULTS AND DISCUSSION : REMDESIVIR

Remdesivir is an antiviral drug which was developed by the biopharmaceutical company Gilead Sciences. Let G_3 be the molecular graph of remdesivir. This graph has 41 vertices and 44 edges.

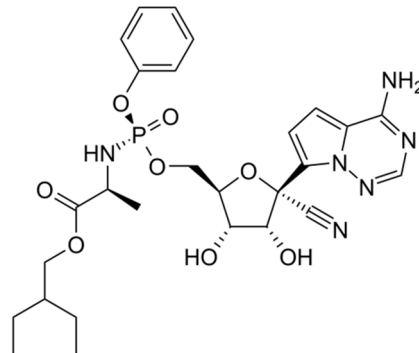


Figure 3. Chemical structure of remdesivir

From Figure 3, we obtain that

- (i) $\{(d(u), d(v)) \mid uv \in E(G_3)\}$ has 8 bond set partitions,
- (ii) $\{(n(u), n(v)) \mid uv \in E(G_3)\}$ has 25 bond set partitions,
- (iii) $\{(m(u), m(v)) \mid uv \in E(G_3)\}$ has 23 bond set partitions,
- (iv) $\{(\varepsilon(u), \varepsilon(v)) \mid uv \in E(G_3)\}$ has 11 bond set partitions,
- (iv) $\{(s(u), s(v)) \mid uv \in E(G_3)\}$ has 23 bond set partitions.

Table 3. Bond set partitions of remdesivir

| | | | | | | | | |
|---------------------------------|---------|---------|---------|---------|---------|---------|---------|---------|
| $d(u), d(v) \mid uv \in E(G_3)$ | (1,2) | (1, 3) | (1, 4) | (2, 2) | (2, 3) | (2, 4) | (3, 3) | (3, 4) |
| Number of bonds | 2 | 5 | 2 | 9 | 14 | 4 | 6 | 2 |
| $n(u), n(v) \mid uv \in E(G_3)$ | (1,6) | (1,34) | (1,38) | (1,39) | (2,37) | (3,12) | (3,23) | (3,36) |
| Number of bonds | 1 | 1 | 2 | 9 | 8 | 1 | 1 | 2 |
| | (4,32) | (4,33) | (4,34) | (4,35) | (5,34) | (6,32) | (6,33) | (8,31) |
| | 1 | 1 | 1 | 1 | 2 | 1 | 2 | 1 |
| | (9,30) | (10,29) | (11,28) | (12,24) | (13,24) | (13,25) | (17,22) | (18,21) |
| | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| | (19,20) | | | | | | | |

| | | | | | | | | |
|---|---------|---------|---------|---------|---------|---------|---------|---------|
| | 1 | | | | | | | |
| $m(u),m(v)\setminus uv\in E(G_3)$ | (1,42) | (1,43) | (2,8) | (2,32) | (2,40) | (2,41) | (3,39) | (4,15) |
| Number of bonds | 2 | 9 | 1 | 1 | 2 | 6 | 2 | 1 |
| | (4,39) | (4,26) | (5,37) | (5,38) | (6,35) | (6,37) | (7,36) | (8,35) |
| | 1 | 1 | 2 | 1 | 1 | 2 | 1 | 2 |
| | (10,33) | (11,32) | (15,27) | (16,26) | (16,27) | (20,23) | (21,22) | |
| $\varepsilon(u),\varepsilon(v)\setminus uv\in E(G_3)$ | 1 | 2 | 1 | 1 | 1 | 1 | 2 | |
| Number of bonds | (9,10) | (10,11) | (11,12) | (12,13) | (13,13) | (13,14) | (14,15) | (15,16) |
| | 2 | 4 | 4 | 7 | 1 | 7 | 5 | 4 |
| | (16,16) | (16,17) | (17,18) | | | | | |
| | 1 | 4 | 5 | | | | | |
| $s(u),s(v)\setminus uv\in E(G_3)$ | (2,4) | (3,6) | (3,7) | (3,8) | (4,4) | (4,5) | (4,6) | (4,7) |
| Number of bonds | 2 | 3 | 1 | 1 | 2 | 4 | 2 | 1 |
| | (4,9) | (5,5) | (5,6) | (5,7) | (5,8) | (5,9) | (6,6) | (6,7) |
| | 1 | 2 | 6 | 1 | 2 | 1 | 1 | 3 |
| | (6,8) | (7,7) | (7,8) | (7,9) | (8,8) | (8,9) | (9,9) | |
| | 1 | 4 | 1 | 1 | 1 | 2 | 1 | |

In the following theorem, we compute the different versions of multiplicative arithmetic-geometric indices of remdesivir.

Theorem 3. Let G_3 be the chemical structure of remdesivir. Then

$$(i) AG_1II(G_3) = \left(\frac{3}{2\sqrt{2}}\right)^2 \times \left(\frac{2}{\sqrt{3}}\right)^5 \times \left(\frac{5}{4}\right)^2 \times \left(\frac{5}{2\sqrt{6}}\right)^{14} \times \left(\frac{3}{2\sqrt{2}}\right)^4 \times \left(\frac{7}{4\sqrt{3}}\right)^2.$$

$$(ii) AG_2II(G_3) = \left(\frac{7}{2\sqrt{6}}\right)^1 \times \left(\frac{35}{2\sqrt{34}}\right)^1 \times \left(\frac{39}{2\sqrt{38}}\right)^2 \times \left(\frac{20}{\sqrt{39}}\right)^9 \times \left(\frac{39}{2\sqrt{74}}\right)^8 \times \left(\frac{5}{4}\right)^1 \times \left(\frac{13}{\sqrt{69}}\right)^1 \\ \times \left(\frac{13}{4\sqrt{3}}\right)^2 \times \left(\frac{9}{4\sqrt{2}}\right)^1 \times \left(\frac{37}{4\sqrt{33}}\right)^1 \times \left(\frac{19}{2\sqrt{34}}\right)^1 \times \left(\frac{39}{4\sqrt{35}}\right)^1 \times \left(\frac{39}{2\sqrt{170}}\right)^2 \\ \times \left(\frac{19}{8\sqrt{3}}\right)^1 \times \left(\frac{13}{2\sqrt{22}}\right)^2 \times \left(\frac{39}{4\sqrt{62}}\right)^1 \times \left(\frac{13}{2\sqrt{30}}\right)^1 \times \left(\frac{39}{2\sqrt{290}}\right)^1 \times \left(\frac{39}{4\sqrt{77}}\right)^1 \\ \times \left(\frac{3}{2\sqrt{2}}\right)^1 \times \left(\frac{37}{4\sqrt{78}}\right)^1 \times \left(\frac{19}{5\sqrt{13}}\right)^1 \times \left(\frac{39}{2\sqrt{374}}\right)^1 \times \left(\frac{13}{2\sqrt{42}}\right)^1 \times \left(\frac{39}{4\sqrt{95}}\right)^1.$$

$$(iii) AG_3II(G_3) = \left(\frac{43}{2\sqrt{42}}\right)^2 \times \left(\frac{22}{\sqrt{43}}\right)^9 \times \left(\frac{5}{4}\right)^1 \times \left(\frac{17}{8}\right)^1 \times \left(\frac{21}{4\sqrt{5}}\right)^2 \times \left(\frac{43}{2\sqrt{82}}\right)^6 \\ + \left(\frac{7}{\sqrt{13}}\right)^2 \times \left(\frac{19}{4\sqrt{15}}\right)^1 \times \left(\frac{43}{4\sqrt{39}}\right)^1 \times \left(\frac{15}{2\sqrt{26}}\right)^1 \times \left(\frac{21}{\sqrt{185}}\right)^2 \times \left(\frac{43}{2\sqrt{190}}\right)^1 \\ + \left(\frac{41}{2\sqrt{210}}\right)^1 \times \left(\frac{43}{2\sqrt{222}}\right)^2 \times \left(\frac{43}{12\sqrt{7}}\right)^1 \times \left(\frac{43}{4\sqrt{70}}\right)^2 \times \left(\frac{43}{2\sqrt{330}}\right)^1 \times \left(\frac{43}{8\sqrt{22}}\right)^2 \\ + \left(\frac{7}{\sqrt{45}}\right)^1 \times \left(\frac{21}{4\sqrt{26}}\right)^1 \times \left(\frac{43}{24\sqrt{3}}\right)^1 \times \left(\frac{43}{4\sqrt{115}}\right)^1 \times \left(\frac{43}{2\sqrt{483}}\right)^2.$$

$$(iv) AG_4II(G_3) = \left(\frac{19}{6\sqrt{10}}\right)^2 \times \left(\frac{21}{2\sqrt{110}}\right)^4 \times \left(\frac{23}{4\sqrt{33}}\right)^4 \times \left(\frac{25}{4\sqrt{39}}\right)^7 \times \left(\frac{27}{2\sqrt{182}}\right)^7$$

$$\begin{aligned} & \times \left(\frac{29}{2\sqrt{210}}\right)^5 \times \left(\frac{31}{8\sqrt{15}}\right)^4 \times \left(\frac{33}{8\sqrt{17}}\right)^4 \times \left(\frac{35}{6\sqrt{34}}\right)^5. \\ \text{(v) } AG_5II(G_3) &= \left(\frac{3}{2\sqrt{2}}\right)^2 \times \left(\frac{3}{2\sqrt{2}}\right)^3 \times \left(\frac{5}{\sqrt{21}}\right)^1 \times \left(\frac{11}{4\sqrt{6}}\right)^1 \times \left(\frac{9}{4\sqrt{5}}\right)^4 \times \left(\frac{5}{2\sqrt{6}}\right)^2 \\ & \times \left(\frac{11}{4\sqrt{7}}\right)^1 \times \left(\frac{13}{12}\right)^1 \times \left(\frac{11}{2\sqrt{30}}\right)^6 \times \left(\frac{12}{2\sqrt{35}}\right)^1 \times \left(\frac{13}{4\sqrt{10}}\right)^2 \times \left(\frac{7}{3\sqrt{5}}\right)^1 \\ & \times \left(\frac{13}{2\sqrt{42}}\right)^3 \times \left(\frac{7}{4\sqrt{3}}\right)^1 \times \left(\frac{15}{4\sqrt{14}}\right)^1 \times \left(\frac{8}{3\sqrt{7}}\right)^1 \times \left(\frac{17}{12\sqrt{2}}\right)^2. \end{aligned}$$

Proof: By using the definitions and cardinalities of the bond partitions of G_3 , we deduce

$$\begin{aligned} \text{(i) } AG_1II(G_3) &= \prod_{uv \in E(G_3)} \frac{d(u)+d(v)}{2\sqrt{d(u)d(v)}} \\ &= \left(\frac{1+2}{2\sqrt{1 \times 2}}\right)^2 \times \left(\frac{1+3}{2\sqrt{1 \times 3}}\right)^5 \times \left(\frac{1+4}{2\sqrt{1 \times 4}}\right)^2 \times \left(\frac{2+2}{2\sqrt{2 \times 2}}\right)^9 \times \left(\frac{2+3}{2\sqrt{2 \times 3}}\right)^{14} \\ & \times \left(\frac{2+4}{2\sqrt{2 \times 4}}\right)^4 \times \left(\frac{3+3}{2\sqrt{3 \times 3}}\right)^6 \times \left(\frac{3+4}{2\sqrt{3 \times 4}}\right)^2. \end{aligned}$$

After simplification, we get the desired result.

$$\begin{aligned} \text{(ii) } AG_2II(G_3) &= \prod_{uv \in E(G_3)} \frac{n(u)+n(v)}{2\sqrt{n(u)n(v)}} \\ &= \left(\frac{1+6}{2\sqrt{1 \times 6}}\right)^1 \times \left(\frac{1+34}{2\sqrt{1 \times 34}}\right)^1 \times \left(\frac{1+38}{2\sqrt{1 \times 38}}\right)^2 \times \left(\frac{1+39}{2\sqrt{1 \times 39}}\right)^9 \times \left(\frac{2+37}{2\sqrt{2 \times 37}}\right)^8 \\ & \times \left(\frac{3+12}{2\sqrt{3 \times 12}}\right)^1 \times \left(\frac{3+23}{2\sqrt{3 \times 23}}\right)^1 \times \left(\frac{3+36}{2\sqrt{3 \times 36}}\right)^2 \times \left(\frac{4+32}{2\sqrt{4 \times 32}}\right)^1 \times \left(\frac{4+33}{2\sqrt{4 \times 33}}\right)^1 \\ & \times \left(\frac{4+34}{2\sqrt{4 \times 34}}\right)^1 \times \left(\frac{4+35}{2\sqrt{4 \times 35}}\right)^1 \times \left(\frac{5+34}{2\sqrt{5 \times 34}}\right)^2 \times \left(\frac{6+32}{2\sqrt{6 \times 32}}\right)^1 \times \left(\frac{6+33}{2\sqrt{6 \times 33}}\right)^2 \\ & \times \left(\frac{8+31}{2\sqrt{8 \times 31}}\right)^1 \times \left(\frac{9+30}{2\sqrt{9 \times 30}}\right)^1 \times \left(\frac{10+29}{2\sqrt{10 \times 29}}\right)^1 \times \left(\frac{11+28}{2\sqrt{11 \times 28}}\right)^1 \times \left(\frac{12+24}{2\sqrt{12 \times 24}}\right)^1 \\ & \times \left(\frac{13+24}{2\sqrt{13 \times 24}}\right)^1 \times \left(\frac{13+25}{2\sqrt{13 \times 25}}\right)^1 \times \left(\frac{17+22}{2\sqrt{17 \times 22}}\right)^1 \times \left(\frac{18+21}{2\sqrt{18 \times 21}}\right)^1 \times \left(\frac{19+20}{2\sqrt{19 \times 20}}\right)^1. \end{aligned}$$

After simplification, we get the desired result.

$$\begin{aligned} \text{(iii) } AG_3II(G_3) &= \prod_{uv \in E(G_3)} \frac{m(u)+m(v)}{2\sqrt{m(u)m(v)}} \\ &= \left(\frac{1+42}{2\sqrt{1 \times 42}}\right)^2 \times \left(\frac{1+43}{2\sqrt{1 \times 43}}\right)^9 \times \left(\frac{2+8}{2\sqrt{2 \times 8}}\right)^1 \times \left(\frac{2+32}{2\sqrt{2 \times 32}}\right)^1 \times \left(\frac{2+40}{2\sqrt{2 \times 40}}\right)^2 \\ & \times \left(\frac{2+41}{2\sqrt{2 \times 41}}\right)^6 \times \left(\frac{3+39}{2\sqrt{3 \times 39}}\right)^2 \times \left(\frac{4+15}{2\sqrt{4 \times 15}}\right)^1 \times \left(\frac{4+39}{2\sqrt{4 \times 39}}\right)^1 \times \left(\frac{4+26}{2\sqrt{4 \times 26}}\right)^1 \end{aligned}$$

$$\begin{aligned}
 & \times \left(\frac{5+37}{2\sqrt{5 \times 37}} \right)^2 \times \left(\frac{5+38}{2\sqrt{5 \times 38}} \right)^1 \times \left(\frac{6+35}{2\sqrt{6 \times 35}} \right)^1 \times \left(\frac{6+37}{2\sqrt{6 \times 37}} \right)^2 \times \left(\frac{7+36}{2\sqrt{7 \times 36}} \right)^1 \\
 & \times \left(\frac{8+35}{2\sqrt{8 \times 35}} \right)^2 \times \left(\frac{10+33}{2\sqrt{10 \times 33}} \right)^1 \times \left(\frac{11+32}{2\sqrt{11 \times 32}} \right)^2 \times \left(\frac{15+27}{2\sqrt{15 \times 27}} \right)^1 \times \left(\frac{16+26}{2\sqrt{16 \times 26}} \right)^1 \\
 & \times \left(\frac{16+27}{2\sqrt{16 \times 27}} \right)^1 \times \left(\frac{20+23}{2\sqrt{20 \times 23}} \right)^1 \times \left(\frac{21+22}{2\sqrt{21 \times 22}} \right)^2.
 \end{aligned}$$

After simplification, we get the desired result.

$$\begin{aligned}
 \text{(iv)} \quad AG_4H(G_3) &= \prod_{uv \in E(G_3)} \frac{\varepsilon(u) + \varepsilon(v)}{2\sqrt{\varepsilon(u)\varepsilon(v)}} \\
 &= \left(\frac{9+10}{2\sqrt{9 \times 10}} \right)^2 + \left(\frac{10+11}{2\sqrt{10 \times 11}} \right)^4 + \left(\frac{11+12}{2\sqrt{11 \times 12}} \right)^4 + \left(\frac{12+13}{2\sqrt{12 \times 13}} \right)^7 + \left(\frac{13+13}{2\sqrt{13 \times 13}} \right)^1 \\
 &+ \left(\frac{13+14}{2\sqrt{13 \times 14}} \right)^7 + \left(\frac{14+15}{2\sqrt{14 \times 15}} \right)^5 + \left(\frac{15+16}{2\sqrt{15 \times 16}} \right)^4 + \left(\frac{16+16}{2\sqrt{16 \times 16}} \right)^1 + \left(\frac{16+17}{2\sqrt{16 \times 17}} \right)^4 \\
 &+ \left(\frac{17+18}{2\sqrt{17 \times 18}} \right)^5.
 \end{aligned}$$

After simplification, we get the desired result.

$$\begin{aligned}
 \text{(v)} \quad AG_5H(G_3) &= \prod_{uv \in E(G_3)} \frac{s(u) + s(v)}{2\sqrt{s(u)s(v)}} \\
 &= \left(\frac{2+4}{2\sqrt{2 \times 4}} \right)^2 + \left(\frac{3+6}{2\sqrt{3 \times 6}} \right)^3 + \left(\frac{3+7}{2\sqrt{3 \times 7}} \right)^1 + \left(\frac{3+8}{2\sqrt{3 \times 8}} \right)^1 + \left(\frac{4+4}{2\sqrt{4 \times 4}} \right)^2 \\
 &+ \left(\frac{4+5}{2\sqrt{4 \times 5}} \right)^4 + \left(\frac{4+6}{2\sqrt{4 \times 6}} \right)^2 + \left(\frac{4+7}{2\sqrt{4 \times 7}} \right)^1 + \left(\frac{4+9}{2\sqrt{4 \times 9}} \right)^1 + \left(\frac{5+5}{2\sqrt{5 \times 5}} \right)^2 \\
 &+ \left(\frac{5+6}{2\sqrt{5 \times 6}} \right)^6 + \left(\frac{5+7}{2\sqrt{5 \times 7}} \right)^1 + \left(\frac{5+8}{2\sqrt{5 \times 8}} \right)^2 + \left(\frac{5+9}{2\sqrt{5 \times 9}} \right)^1 + \left(\frac{6+6}{2\sqrt{6 \times 6}} \right)^1 \\
 &+ \left(\frac{6+7}{2\sqrt{6 \times 7}} \right)^3 + \left(\frac{6+8}{2\sqrt{6 \times 8}} \right)^1 + \left(\frac{7+7}{2\sqrt{7 \times 7}} \right)^4 + \left(\frac{7+8}{2\sqrt{7 \times 8}} \right)^1 + \left(\frac{7+9}{2\sqrt{7 \times 9}} \right)^1 \\
 &+ \left(\frac{8+8}{2\sqrt{8 \times 8}} \right)^1 + \left(\frac{8+9}{2\sqrt{8 \times 9}} \right)^2 + \left(\frac{9+9}{2\sqrt{9 \times 9}} \right)^1.
 \end{aligned}$$

After simplification, we get the desired result.

5. CONCLUSION

In this paper, we have computed the first, second, third, fourth and fifth multiplicative arithmetic-geometric indices of some important chemical drugs which appeared in medical science.

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